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A. Huba

Problems of dynamical modelling of mechatronic motion transducers

Abstract: *One of the most frequently used motion transformers in mechatronics is the ball screw. The right dynamic model and the correct worth of the resulting screw stiffness are demanded for the calculation of the control system. The paper shows practical method for the dynamical modeling based on the network theory. The descriptions and modeling of control of precision positioning systems often neglect the flexibility of mechanical elements and the model of the plant is represented only as a first order system in form of a mass or inertia. This paper shows the indefensibility of this conception for precision devices.*

Keywords*: Mechatronic motion transducer, dynamical modeling of ball screw, resulting stiffness.*

1. ENERGY TRANSFORMERS IN MECHATRONICS

Ball screw and tooth belt motion transducers belong to the group of energy transformers in the mechatronics since it couples two different physical system types to each other while the transformer variables belonging to the different systems are the same type. This type of energy transformers are called as “transducer”. The motion transformers are important subsystems in complex mechatronic devices. The problem appears frequently that high speed linear motion for believed length with extreme accuracy is desired. There is recently however no suitable linear actuator on the market to realize the parameters of these demands. Usually DC servo motors are applied as actuators coupled with a gear box to increase the torque and to realize the demanded angle speed and finally a special tooth belt system or a ball screw motion transducer ensure the desired linear motion.

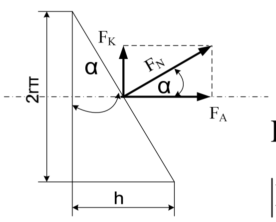
Like others also this energy transformer is only ideal loss free [1], [3]. There are beside the dissipative elements energy storage elements in the system too and the system order responsible for the dynamical behavior is depending on the actual construction. How near to the ideal loss free system the ball screw lays show the degree of efficiency which is as high as 96 % thanks the low friction. The transformer constant is determined by the geometrical characteristics of the ball screw.

1.1. TRANSFORMER CONSTANT OF THE BALL SCREW

As mentioned the energy dissipative elements will be taken into consideration together with other passive elements of the coupled systems. We regard the transformer as loss free and can write:

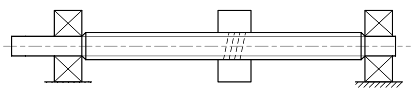
$$\begin{aligned} P_{rot} &= P_{lin} \\ \Omega(t)\cdot M(t) &= f(t)\cdot v(t) \end{aligned} \tag{1}$$

The Fig. 1. shows the geometrical parameters and the force components needed for the calculation of the transformer constant. In the Fig. 2. one sees the graph of the transformer and the direction of the elements. According the definitions both graph elements are directed in opposite direction, and the equation there is a negative sign for the through variables.



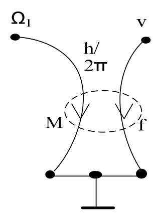
$$F_A \,\mathrm{tg}\alpha = F_K$$

$$|M| = rF_K = rF_A \,\mathrm{tg}\alpha = rF_A \frac{h}{2r\pi} = F_A \frac{h}{2\pi}$$



$$\frac{x}{\varphi} = \frac{v}{\Omega} = \frac{h}{2\pi}$$

Fig. 1. Determination of the transfer constant from the geometry



$$M = -\frac{h}{2\pi} f$$

$$\Omega = \frac{2\pi}{h} v$$

$$n_v = \frac{h}{2\pi}$$

Fig. 2. Graph and equations of the transducer

1. 2. RESULTING ELASTICITY IN CATALOGUES

The knowledge of the dynamical model is essential for the effective and appropriate design process of controlled positioning systems including the simulations and respectively the non linear system elements.

The equilibriums and calculation forms published in firm product catalogues are not always usable without control of applicability especially for the dynamical simulations. The technical control by measuring is sometimes complicated however the sure parameter estimation is possible only by measuring. One uses the data base of catalogues normally before the purchasing of the elected construction elements for example exactly to simulate the behavior by computer. Especially for purpose of dynamical simulations the network theory is useful offering a complex method to describe interdisciplinary systems. Using this method one does not substitute simply the parameters in prepared equations but analyses consciously the given technical structure and applies physical laws. The structure analysis by graphs or in linear systems by impedances let see every relevant details of a given system with the necessary accuracy.

Let see one characteristic example cited from a product catalogue of a well known firm which has to stay anonym. The resulting stiffness of the whole ball screw system can be calculated according this description as shown in the Fig. 3.

Gesamtsteifigkeit C_{ges} des Kugelgewinde-Systems

Die Gesamtsteifigkeit C_{ges} des Kugelgewinde-Systems wird aus folgender Gleichung ermittelt:

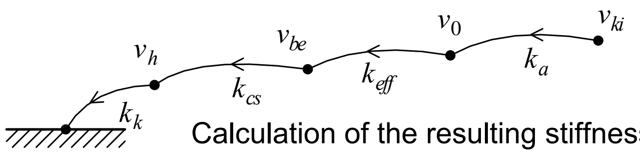
$$\frac{1}{C_{ges}} = \frac{1}{C_{mc}} + \frac{1}{C_{Sp}} + \frac{1}{C_{L}} + \frac{1}{C_{U}}$$

Fig.3. One characteristic catalogue formula for the calculation of resulting stiffness

The equation and the units show that the parameter C_i denotes the stiffness of the main component of the complex mechanical system and also show that these model elements can be connected only in series. The unit shows that the stiffness is interpreted as a linear motion parameter and not as torsion stiffness in each case.

Based on the formula the imagination of the sophisticated system structure is reproducible. We use normally “k” (translational) and “K” (rotational) instead of “C” to denote the stiffness in linear motion systems so we do it next also in this paper. To discriminate the stiffness in rotation systems we use “K”.

Resulting „k” seen from point of the local variable $v_{ki}(t)$ (velocity)



Calculation of the resulting stiffness

$$\frac{1}{k_e} = \frac{1}{k_a} + \frac{1}{k_{eff}} + \frac{1}{k_{cs}} + \frac{1}{k_k}$$

k_{min} is dominant

Fig. 4. The resulting stiffness based on the formula of the catalogue

2. STEPS OF THE CORRECT NETWORK MODELLING

The serial connection of the elastic elements is suitable if the resulting stiffness between the nut and the frame construction is to determinate, shown in the Fig. 4. The task of analysis may be also this if the stiffness of the mechanical system is looked for. However this model can not be used for the purpose of control system design. In positioning systems the motor-gearbox unit is directly connected by a coupling to the ball screw (local variable point “v_{be}” in the Fig. 4.). Following this the angle velocity input of the ball screw transducer is connected to this point. By the way as “true” variable for rotational mechanical systems would be better to use the angle velocity (Ω) but in this paper consequently the denoting of the catalogues will be used. The transformation (reduction) of the rotation parameter into linear motion parameter is no problem using the equilibriums of the Fig. 2. as we show it later.

The torsion stiffness of the coupling is usually smaller in real technical systems first of all in precision positioning devices according this it is reasonable not to neglect this. On the other hand this further elastic element does not mean a new independent energy storage function since there is a possibility for reducing depending on the structure.

Identification of the independent cross variables (local variables)

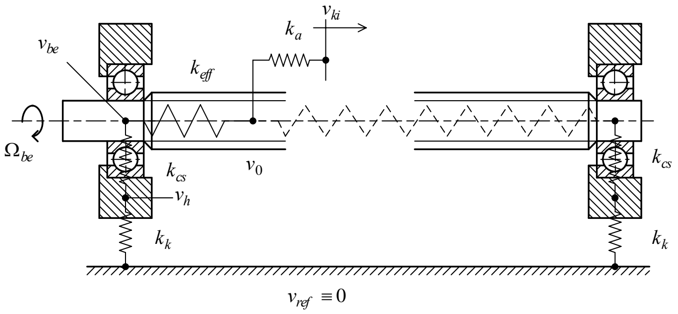


Fig. 5. The functional structure and the four elastic elements with representing of the drive in point

But it is important to say in connection with this modeling problem that the descriptions and modeling of control of precision positioning systems often neglect the flexibility of mechanical elements and the model of the plant is represented only as a first order system in form of a mass or inertia. This paper shows the indefensibility of this conception in the case of precision devices.

The Fig. 5 shows the functional model and the elastic elements according to the catalogue. The first step in the network modeling process is the identification of the independent across variables, in the given system these are the velocities.

System graph of the ball screw without the coupling elasticity if one calculates with reduced linear motion elements suggested by the catalogue

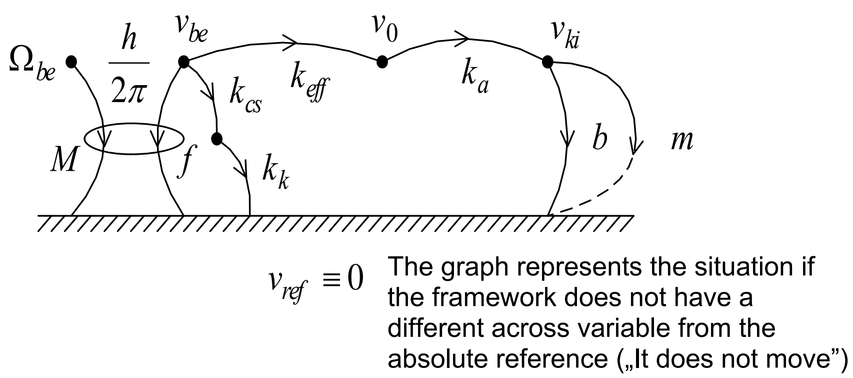


Fig. 6. Graph of ball screw with reduced linear elements and without coupling elasticity

$$\frac{1}{k_{e1}} = \frac{1}{k_{cs}} + \frac{1}{k_k} \longrightarrow k_{e1} = \frac{k_{cs} \cdot k_k}{k_{cs} + k_k} \qquad k_k > k_{cs}$$

$$\frac{1}{k_{e2}} = \frac{1}{k_a} + \frac{1}{k_{eff}} \longrightarrow k_{e2} = \frac{k_a \cdot k_{eff}}{k_a + k_{eff}} \qquad k_{eff} > k_a$$

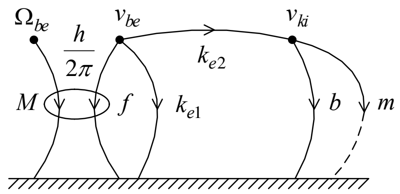


Fig. 7. Simplified graph of the system shown in Fig. 6.

If all relevant across variables are identified one may sketch the structure graph see Fig. 6. This system graph of the ball screw does not consider the coupling elasticity and it calculates with reduced linear motion elements suggested by the catalogue. It is very important further that the graph represents the situation if the framework does not have a different across variable from the absolute reference I that means the framework does not move compared to the absolute reference (gravitation system). In the figures Ω_{be} means the input angle velocity and there is also the motion transformer to see. The output across variable is the velocity of the mass, i.e. the mass of the driven table. We also showed the linear damping coefficient “b” of the guiding. This model element is in the reality of course not a linear one. The non linear behavior can be taken into consideration better after completing the motion equilibrium first of all if one creates a state model for digital simulation.

Fig. 7. shows that the resulting stiffness of the serial elastic elements can be calculated and the graph will be more simple. Of course it depends always on the given situation and device to decide which of these elastic elements are relevant for the modeling. Universal solution does not exist. The modeling process supposes some knowledge.

3. THE SIMPLIFICATED DYNAMIC MODELL

Normally the ball screw system is coupled by a special coupling to the output of the gearbox. The construction of this coupling eliminates the errors of angle and axle shifting and beside of this it has suitable torsion stiffness. The first task is to decide if in the given system one has to calculate with the torsion elasticity of this coupling related to the other mechanical elements or not. If there is a danger of resonance with the mass load it is better not to neglect this elasticity. The elasticity of the spindle is almost in every case greater than this of the coupling but the elasticity of the nut is comparable. The Fig. 8. shows the graph completed by the coupling as rotational elastic element.

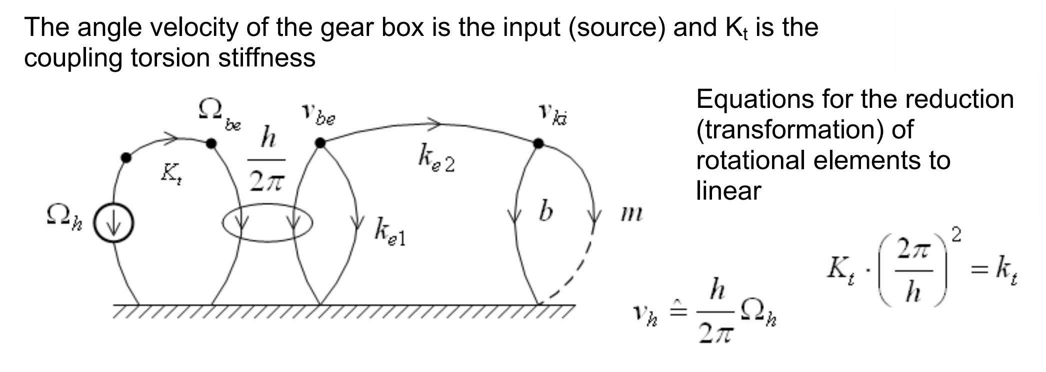


Fig. 8. Linear graph decomposing truly the relevant mechanical elements for the control system design

The network theory helps to reduce the impedances (admittances) of different types of subsystems to each other [2]. There are no further equations needed as the two physical ones for this transformation, shown in the four-pole see Fig. 2. Let write these equations in the following from and the logic of transformation is immediately to understand:

$$Z_{transzl} = \frac{V(s)}{F(s)} = \frac{h/2\pi}{|2\pi/h|}\frac{\Omega(s)}{M(s)} = \left(\frac{h}{2\pi}\right)^2 Z_{rot} \tag{2}$$

The transformation of the source goes on the same way, using the actual physical law.

$$V_h(s) = \frac{h}{2\pi}\Omega_h \tag{3}$$

The first graph of the Fig. 8 shows the network after reduction of the rotational subsystem. There are three characteristic ways to calculate the abstract mathematic model:

- Nodal or loop analysis to get the differential equation
- Laplace transform and impedance method to get the transfer function
- Nomination of state variables for the most universal state model

We choose the impedance method for the further calculations since the transfer function offer the best form for the preliminary stability design of the complete control system.

In the left side of the Fig. 8 we assumed that the stiffness of the bearing and of the framework is bigger than this of the spindle and of the nut. This is in a lot of cases the reality. The right side shows the calculation steps if the stiffness of all elements is to be taken into consideration since these are in the same range. One sees the resulting simplified cross variable attenuator does not differ in the shape only in the resulting stiffness. The difference could be not so important in the first moment however the importance of this is relative high because of the characteristic frequency (resonance) of the Bode diagram, resulting from the main system parameter. This characteristic frequency and the actual system damping determine the type and the parameters of the suitable controller.

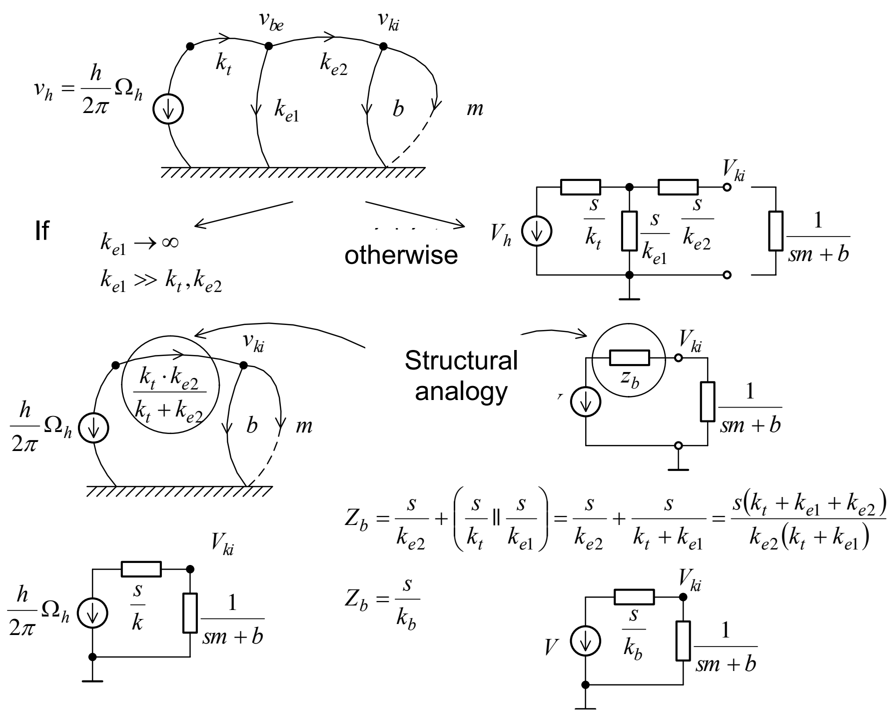


Fig. 8. Transfer function calculation using impedance method

Let write the transfer function looked for at the beginning with help of a cross variable attenuator rule, this is the most simple way since the work was already done by the simplification of the original network. The disadvantage of this “black-box” method is that we can not have a look “into” the system, one sees only the changing of the input and output variables:

$$\frac{V_{ki}}{V_{be}}(s)=\frac{V_{ki}}{\Omega_h \frac{h}{2\pi}}=\frac{\frac{1}{sm+b}}{\frac{s}{k}+\frac{1}{sm+b}}=\frac{k}{s^2m+sb+k}=\frac{1}{s^2\frac{m}{k}+s\frac{b}{k}+1} \tag{4}$$

In the last step we substitute the transformed input with the origin one using the known physical low, and get finally the linear transfer function of the ball screw motion transformer. The second form is usual in the control engineering showing the time constant "T" related to the resonance frequency and the damping "ξ" responsible for the inclination of oscillation.

$$\frac{V_{ki}}{\Omega_h}=\frac{\frac{h}{2\pi}}{s^2\frac{m}{k}+s\frac{b}{k}+1}=\frac{\frac{h}{2\pi}}{s^2T^2+s2\xi T+1} \tag{5}$$

In the case if the damping factor "b" of the guiding or one of the elastic elements can not be modeled with linear parameters one have to continue the calculations using the state model. This linear transfer function helps in that case in the estimation of the control stability around the working points.

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